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On the integral properties of separated laminar boundary layers

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Detailed measurements of the flow over a compression corner were taken using hot-wire probes. The experiments were performed in supersonic flow ($M = 2{\cdot}64$) under adiabatic wall conditions. The incompressible analogues of the boundary-layer profiles were obtained and their integral characteristics and shape factors correlated. Comparison with the self-similar profiles used by Lees & Reeves (1964) to describe interaction problems showed some similarities between the shape factors, but the measured negative shears in the separated bubble proved to be much less.

1. Introduction

At present most experimental data on separated supersonic flows consist of wall pressure measurements. Pressure distributions of this kind have been produced by several investigators: Chapman, Kuehn & Larson (1958) succeeded in correlating pressure distributions up to the so-called plateau pressure; Curle (1961) extended this correlation to include heat transfer effects; and a further extension to cover hypersonic flows was developed by Lewis (1967).

These pressure distributions are accepted as the standard means available for testing solutions obtained theoretically. Of the many theoretical attempts few are successful in predicting the correct pressure distribution; yet, when these methods have been extended to cover non-adiabatic cases, several difficulties have appeared. For this reason, it is believed that experimental descriptions of the flow field giving more information about the boundary-layer profiles in the interaction region are needed to help the theoretician choose proper assumptions while developing his analysis.

In the present note, emphasis was put on detailed measurement of the laminar boundary-layer flow over a compression corner at $M = 2{\cdot}64$ and Re in the range $1{\cdot}3 \times 10^5$ to $1{\cdot}7 \times 10^5$ with compression angles of 9°, 11° and 13°.

2. Experimental procedure

Measurements of the boundary-layer profiles were taken using hot-wire anemometry. This technique is particularly suitable for separated flows owing to the directional insensitivity and minimal disturbances of hot-wire probes. In an earlier attempt by Sfeir (1969) measurements obtained with a hot-wire anemometer and a pitot probe at the same point of a flow were used simultaneously to compute all the dynamic and thermodynamic variables at that point. This, in fact, permitted testing of the constancy of p across the shear layer, which was confirmed for small compression angles. For this reason, in the present report the hot-wire data supplemented by the wall pressure were used to compute the velocity u, temperature T and mass flux profiles in the shear layer.

The hot wire measures essentially two quantities: the resistance at equilibrium (unheated wire) and the heat loss of the wire when it is overheated with respect to the flow around it. These two measurements may be expressed in another form, namely, as the temperature T_{em} at equilibrium and the Nusselt number Nu_m. These have to be corrected for end losses, since the wire has a finite length and is soft-soldered at each end to two needle supports. Two correction factors C_N and C_R can be determined such that

$$Nu = C_N Nu_m, \quad T_e = C_R T_{em}.$$

Nu and T_e are respectively the Nusselt number and recovery temperature of an equivalent wire of infinite length. The correction factors are given by Kovasznay (1953).

Empirical laws established by Dewey (1965) give the following relations for heat loss from infinite cylinders normal to a stream:

$$Nu_0(M, Re_0) = Nu_0(Re_0, \infty)\, \phi(Re_0, M),$$
$$\eta^* = (\eta - \eta_c)/(\eta_f - \eta_c).$$

$Nu_0(Re_0, \infty)$ is a function describing the variation of Nu_0 with Re_0 for $M \gg 1$ and $\phi(Re_0, M)$ is the departure from this relation when M is no longer very large. Nu_0 and Re_0 are, respectively, the Nusselt and Reynolds numbers based on the wire diameter, viscosity and thermal conductivity at local stagnation conditions. η is the ratio of T_e to the total temperature; η_c and η_f are respectively the continuum limit of η and its free molecular limit. It is known that η^* is a unique function of the Knudsen number Kn_∞ for a given M.

Given an additional measurement, either the total or the static pressure, it is possible to reduce the hot-wire data using these relations. An iteration scheme involving the equations for end losses, recovery temperature and heat losses yields the velocity, specific mass and temperature at the point of measurement.

This computational procedure converges quite fast: a few hundred measurements may be reduced on any small-size computer within a very reasonable time. In all, about fifty boundary-layer profiles were measured for the three compression angles tested.

Particular care was taken to achieve two-dimensional laminar-flow conditions throughout the tests. It was found, by mounting side plates at equal distances from the centre-line and by varying this distance, that the flow tends to a limit when the aspect ratio becomes of order one and above, provided that the compression angle is less than 15°. Lewis showed that when this limit is attained two-dimensional conditions prevail. The influence of the ramp length after the compression corner was also thoroughly investigated. In all the measurements reported, this length was larger than the critical length, mentioned by Siriex, Mirande & Delery (1966), below which downstream conditions influence reattachment. These details, as well as a description of the models tested, the probes used and the wind tunnel where the tests were run, are given by Sfeir (1969). This report also includes the relations mentioned above and the numerical procedure for the reduction of the hot-wire measurements.

3. Results and discussions

Several integral methods have had varying success in describing interactions between shocks and laminar boundary layers. One is the moment of momentum method suggested by Sutton (1937) and Tani (1954), and later improved by Lees & Reeves (1964), who used curve-fit relations for all velocity-dependent integral parameters based on Cohen & Reshotko's (1956) similar profiles including the analogues of Stewartson's (1954) 'lower branch' for separated flows. Holden (1970) has extended Lees & Reeves' method to include the energy equation. These methods are in a sense similar to the procedure followed by Thwaites (1949). However, Thwaites's curve-fit is based on a variety of exact solutions and experimental results and is used up to the point of separation. In contrast, the profiles used by Lees & Reeves, in addition to being independent of the pressure-gradient parameter, are not agreed upon as being sufficient or even truly representative of separated boundary layers in such interaction problems. In order to examine this point the incompressible analogues of our measured profiles are found using Stewartson's transformation:

$$dX = \frac{P_e C_e}{P_\infty C_\infty} dx, \quad dY = \frac{C_e \rho}{C_\infty \rho_\infty} dy, \quad U = \frac{C_\infty}{C_e} u,$$

where C is the speed of sound and the subscript e refers to conditions at the edge of the boundary-layer, the subscript ∞ referring to upstream infinity.

The transformed displacement thickness δ_1^*, momentum thickness δ_2^* and mechanical-energy thickness δ_3^* are then found:

$$\delta_1^* = \int_0^{\delta^*} \left(1 - \frac{U}{U_e}\right) dY, \quad \delta_2^* = \int_0^{\delta^*} \frac{U}{U_e}\left(1 - \frac{U}{U_e}\right) dY, \quad \delta_3^* = \int_0^{\delta^*} \frac{U}{U_e}\left(1 - \frac{U^2}{U_e^2}\right) dY,$$

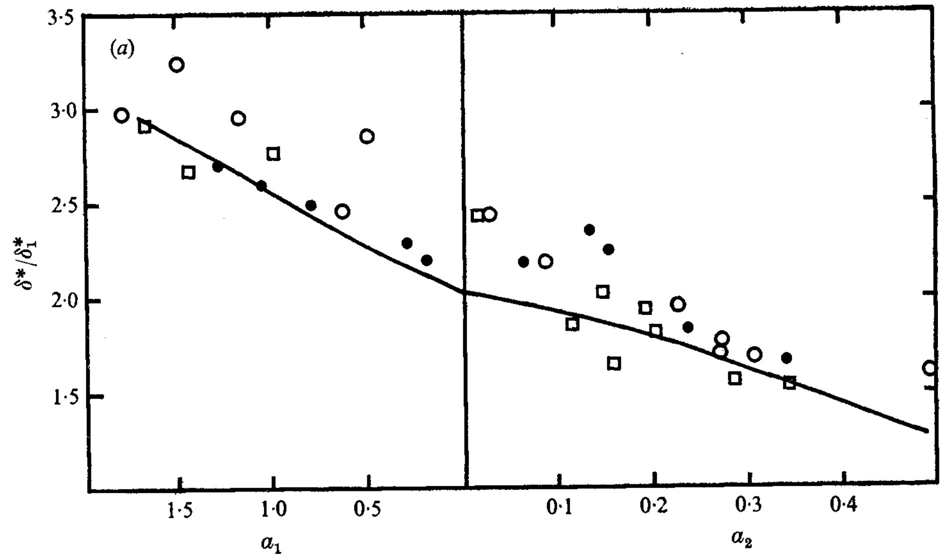
where δ^* is the boundary-layer thickness of the incompressible analogue.

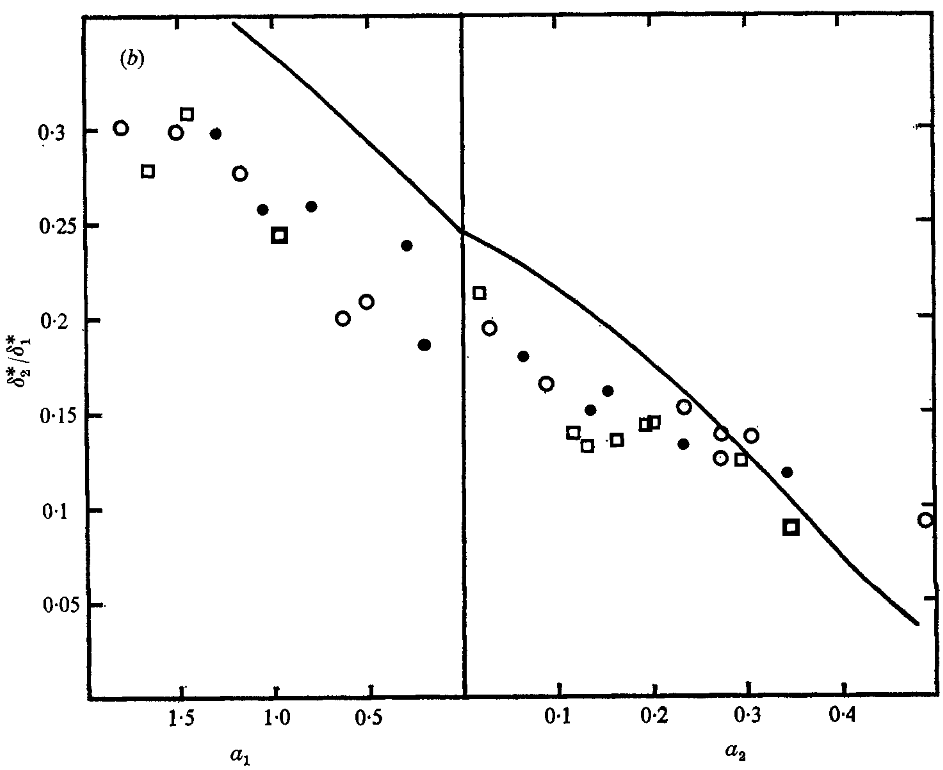
As mentioned earlier in Lees & Reeves's method, the similar profiles are independent of the pressure-gradient parameter and their properties are represented in terms of the velocity parameters a_1 and a_2 defined by

$$a_1 = (\delta^*/U_e)\,[\partial U/\partial Y]_{Y=0} \quad \text{for the attached flow},$$
$$a_2 = [Y/\delta^*]_{U=0} \quad \text{for the separated flow}.$$

a_1 is the normalized transformed velocity gradient at the wall and a_2 is the normalized transformed distance from the wall to the zero-velocity point.

Figures 1 (*a*), (*b*) and (*c*) show δ^*/δ_1^*, δ_2^*/δ_1^* and δ_3^*/δ_1^* as functions of a_1 and a_2 along with the theoretical curve-fits of Lees and Reeves. Although the ranges of variation for the compression angle θ and *Re* are quite small, no trend for any systematic shift of data is observed and a fairly good correlation is obtained for these shape factors in terms of a_1 and a_2. It is to be noted that the data shown are representative of the flow through the whole separation and reattachment pro-





FIGURES 1(a, b). For legend see next page.

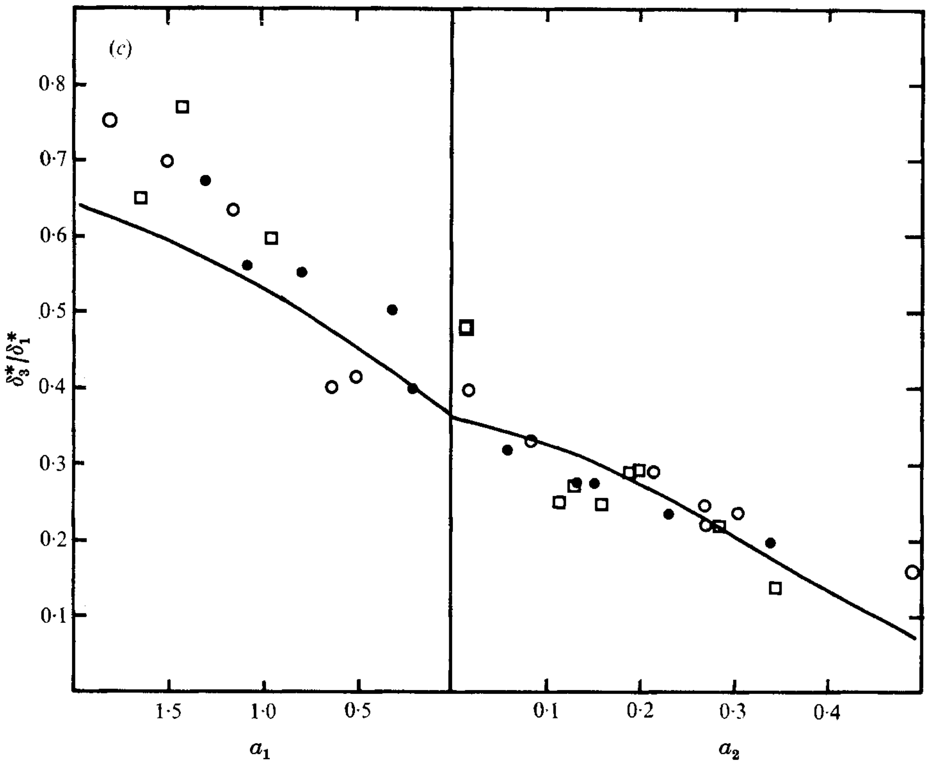


FIGURE 1. (*a*) δ^*/δ_1^*, (*b*) δ_2^*/δ_1^* and (*c*) δ_3^*/δ_1^* as functions of a_1 and a_2; $M = 2{\cdot}64$. ——, curve-fit based on self-similar profiles; □, $\theta = 9°$, $Re = 1{\cdot}40 \times 10^5$; ○, $\theta = 11°$, $Re = 1{\cdot}61 \times 10^5$; ●, $\theta = 13°$, $Re = 1{\cdot}70 \times 10^5$.

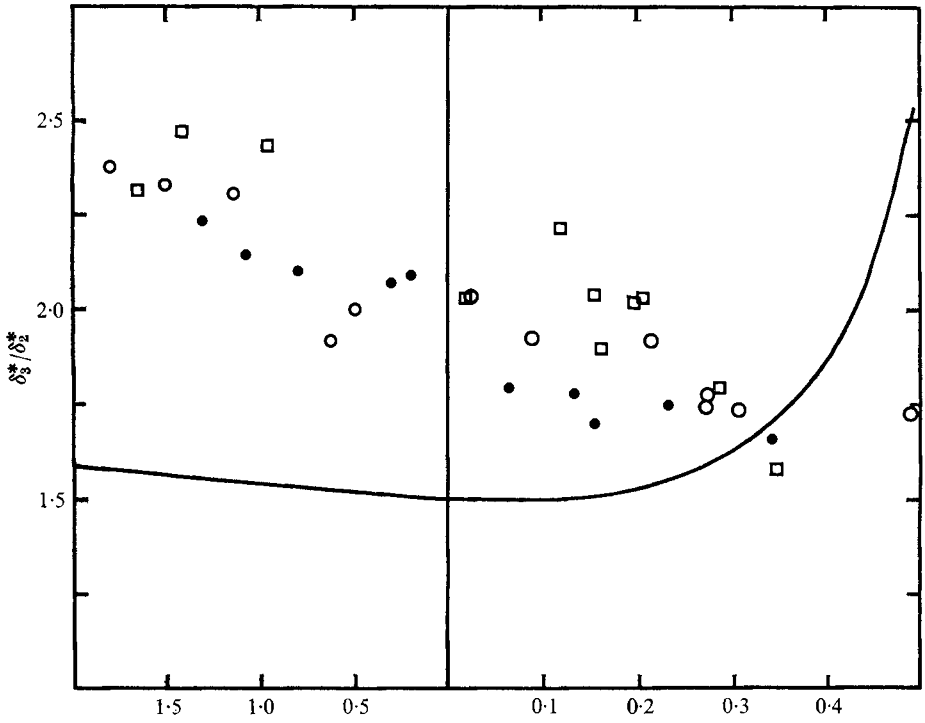


FIGURE 2. δ_3^*/δ_2^* as a function of a_1 and a_2; $M = 2{\cdot}64$. ——, curve-fit based on self-similar profiles; □, $\theta = 9°$, $Re = 1{\cdot}40 \times 10^5$; ○, $\theta = 11°$, $Re = 1{\cdot}61 \times 10^5$; ●, $\theta = 13°$, $Re = 1{\cdot}70 \times 10^5$.

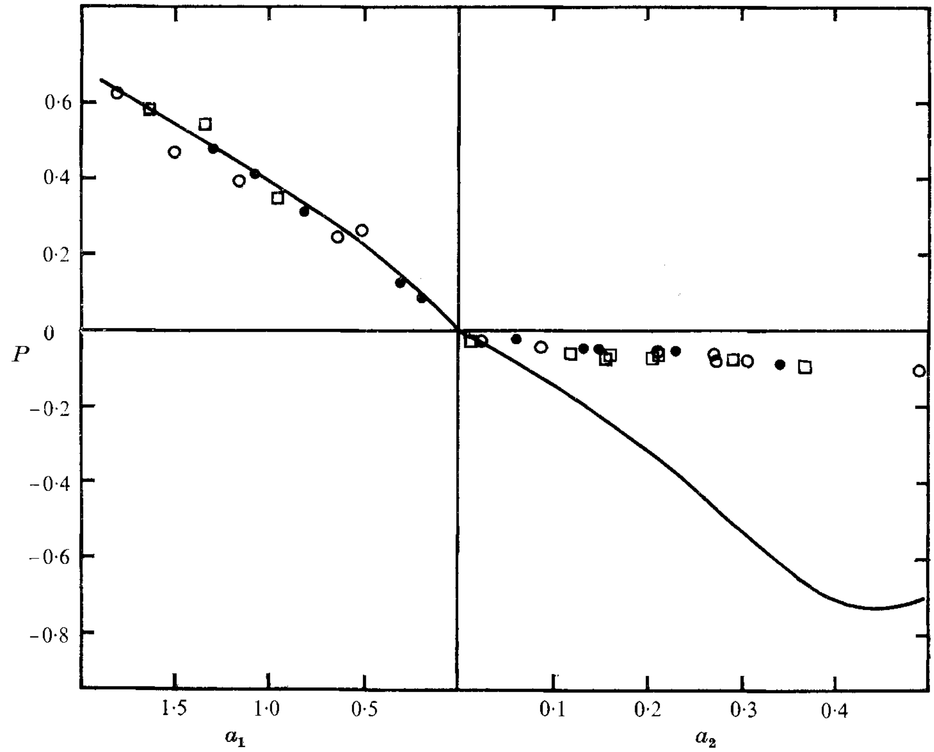


FIGURE 3. P as a function of a_1 and a_2; $M = 2{\cdot}64$. ——, curve-fit based on self-similar profiles; □, $\theta = 9°$, $Re = 1{\cdot}40 \times 10^5$; ○, $\theta = 11°$, $Re = 1{\cdot}61 \times 10^5$; ●, $\theta = 13°$, Re $= 1{\cdot}70 \times 10^5$.

cesses and not only in the 'free interaction' region where Chapman's correlation (Chapman *et al.* 1958) and its extensions are known to hold.

The ratio δ_3^*/δ_2^* is shown in figure 2. Here the theoretical curve-fit based on similar solutions is practically constant for attached flow and increases rapidly with a_2 after separation. This behaviour is quite clearly contradicted by our experimental results. The fact that for similar profiles δ_3^*/δ_2^* is a weak function of a_1 has successfully been used by Hankey & Cross (1967) to obtain a simple and elegant closed-form solution to interaction problems; it would now seem that such an assumption is only reasonable for weak interaction problems with no appreciable separated bubbles.

The largest discrepancy with the similar profiles is to be seen in figure 3, showing the parameter $P = (\delta_1^*/U_e)\,[\partial U/\partial Y]_{Y=0}$ versus a_1 and a_2. This parameter is representative of the wall shear. Good agreement is observed in the attached-flow region. In the separated region the self-similar profiles differ from our results in that they indicate exceedingly large negative shear stresses. Also, the theoretical profiles used by Lees & Reeves have the property that the magnitude of the maximum reversed velocity after separation increases, reaches a maximum, then decreases again with increasing a_2; the opposite occurs during reattachment and a similar variation is followed by the wall shear stress. This complicated sequence of events is not confirmed by our results, where the maximum reversed velocity is found to be a monotonic function of $[Y]_{U=0}$. Two recent studies by

Stewartson & Williams (1969) and Lu (1970) give a correct qualitative description of the reversed velocity although they still indicate larger negative shears than are actually measured.

4. Conclusions

The experimental results shown clearly demonstrate that the transformed shape factors of the measured profiles may be correlated in terms of a_1 and a_2 and are close to the curve-fit based on self-similar solutions. This, however, does not imply a close similarity between our measurements and the theoretical profiles; in fact there is a marked difference, particularly for the reversed flow in the separation region, where the results of Stewartson & Williams and Lu are more representative. It can also be argued that a proper solution, in terms of predicting pressure distribution on an adiabatic wall, may be obtained using profiles with integral properties close to those obtained experimentally, but having different reversed-flow characteristics. In other words, the reversed flow seems to play a negligible part in momentum transfer.

It would seem that part of the difficulties encountered theoretically in non-adiabatic interaction problems may be explained as follows: heat exchange in the separated bubble is a strong function of the flow there; therefore, in order to predict heating or cooling effects accurately, it is necessary that the profiles used, in addition to having the right integral properties, must also properly describe the reversed-flow region. This problem may be resolved, it seems, by including the energy equation as suggested by Holden. Direct comparison with the curve-fits he used is, however, not possible as our data are valid for adiabatic wall conditions only.

More measurements covering a wider range for θ, M and Re are of course necessary to confirm the ideas presented here, particularly for the correlation of δ^*/δ_1^*, δ_2^*/δ_1^* and δ_3^*/δ_1^*. Curve-fits based on experimental profiles may then yield better solutions for interaction problems with heating and cooling.

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